



Politechnika
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Equation of State for astrophysical simulations - current challenges and future perspectives

Anil Kumar

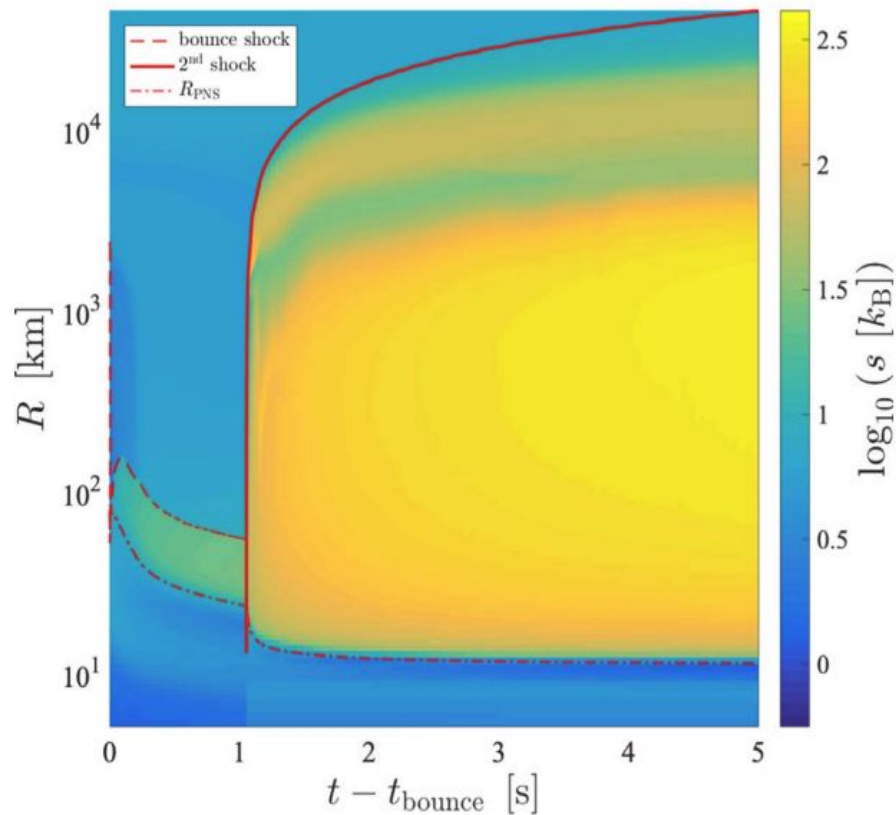
Brainstorming Workshop on
Multimessenger astrophysics with core-collapse supernovae and binary neutron star mergers

Outline

- QCD phase transition driven supernova explosion
- State of the Art for phase transition
- Phase transition with modified excluded volume
- Pions contribution in EOS
- Future perspectives
- Summary

Shock Revival with phase transition

- 1st shock stalls after core bounce
- phase transition in the proto-neutron star can generate 2nd shock
- 2nd shock can revive 1st shock
- The properties of the 2nd shock depend on the phase-transition characteristics



State of the Art

- RDF models

$$\Omega = \Omega_{\text{nucleons}} + \Omega_{\text{nuclei}} + \Omega_{\text{quarks}} + \Omega_{\text{leptons}} + \Omega_{\text{photons}}$$

$$\Omega_{\text{quarks}} = - \sum_i g_i T \int \frac{d^3 p}{(2\pi)^3} \left[\ln \left(1 + e^{-(E_i^* - \mu^*)/T} \right) + \ln \left(1 + e^{-(E_i^* + \mu^*)/T} \right) \right] + U_{\text{eff}}$$

$$U_{\text{eff}} = U(n_s, n_v) - \frac{\partial U}{\partial n_s} n_s - \frac{\partial U}{\partial n_v} n_v$$

$$U(n_s, n_v, n_{vi}) = C n_s^{4/3} + D(n_v) n_s^{2/3} + \omega_4 n_v^2 + \frac{\omega_8 n_v^4}{1 + \omega'_8 n_v^2} + \rho_4 n_{vi}^2$$

Limitations of string flip model

- String flip model has many parameters

$$U(n_s, n_v, n_{vi}) = Cn_s^{4/3} + D(n_v)n_s^{2/3} + \omega_4 n_v^2 + \frac{\omega_8 n_v^4}{1 + \omega'_8 n_v^2} + \rho_4 n_{vi}^2$$

- The density dependence of interactions

$$U \propto n^2 \rightarrow (k_F \ll \mu_{IR})$$

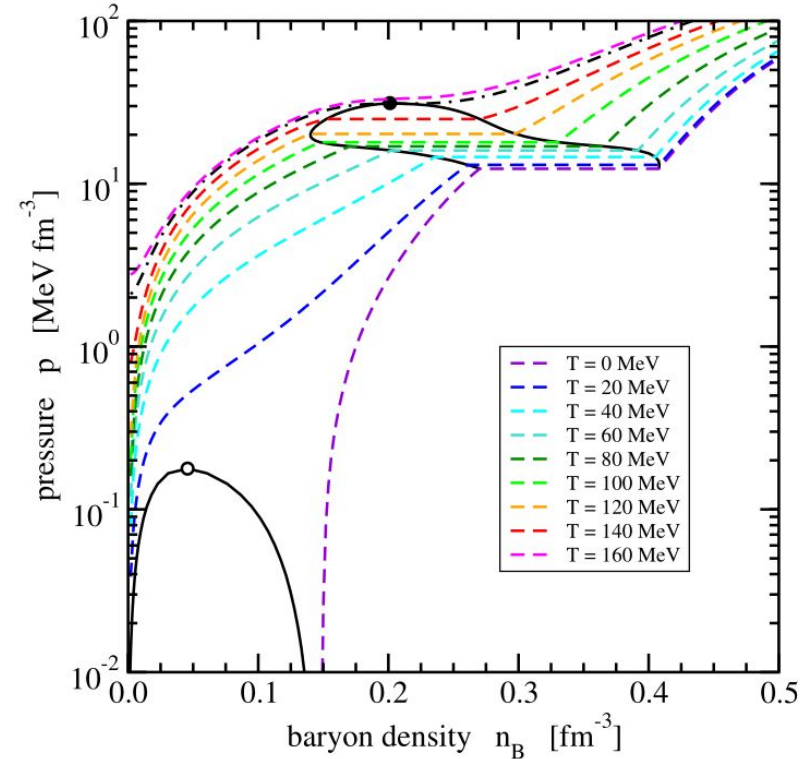
$$U \propto n \rightarrow (k_F \gg \mu_{IR})$$

Van der Waal behaviour in QCD phase transition

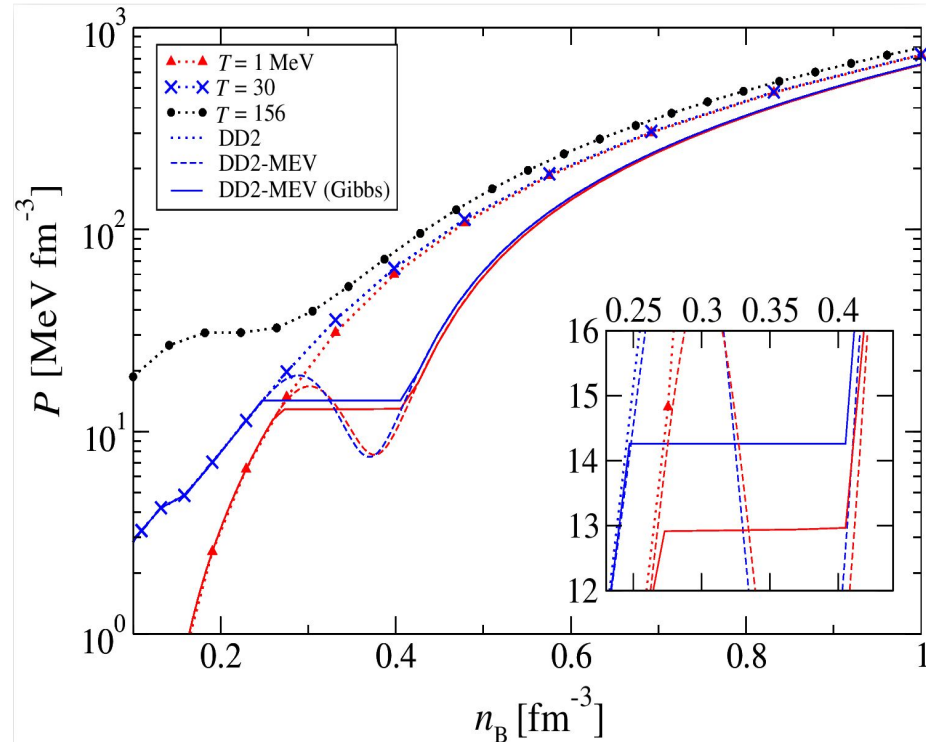
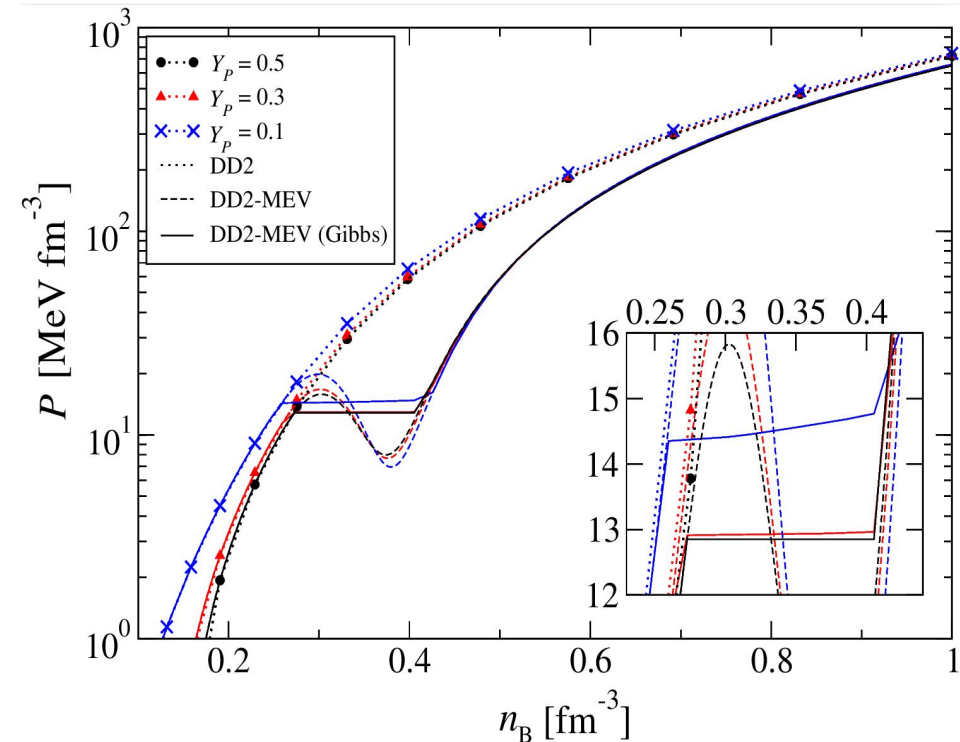
$$\Phi_N(T, x) = 1 + \mathcal{S} g_1(T) \Theta(x) \exp\left(-\frac{1}{2x^2}\right)$$

$$x = v(n_p^{(v)} + n_n^{(v)} - g_8(T) n_{cut})$$

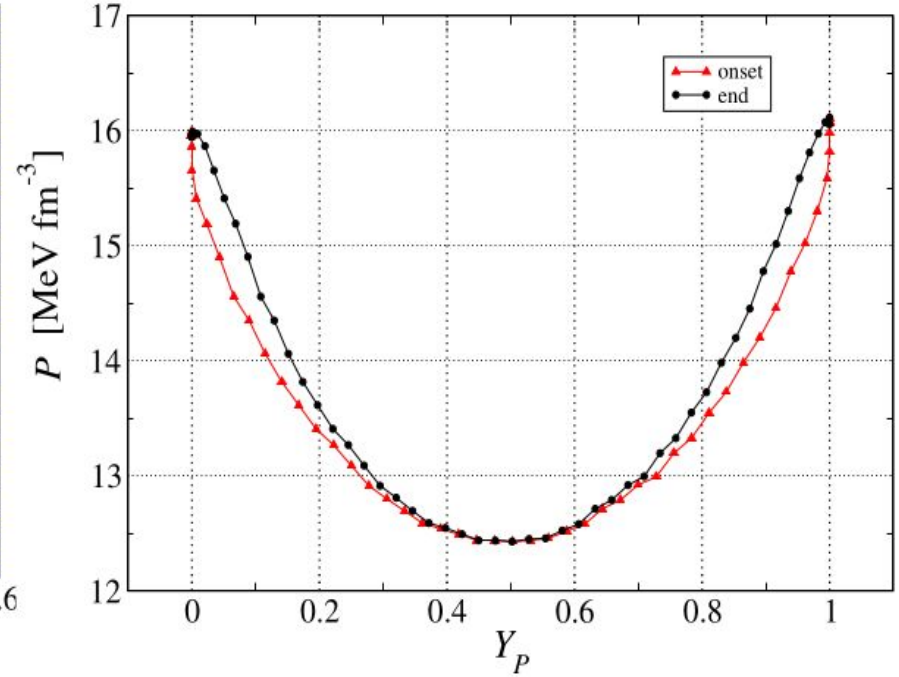
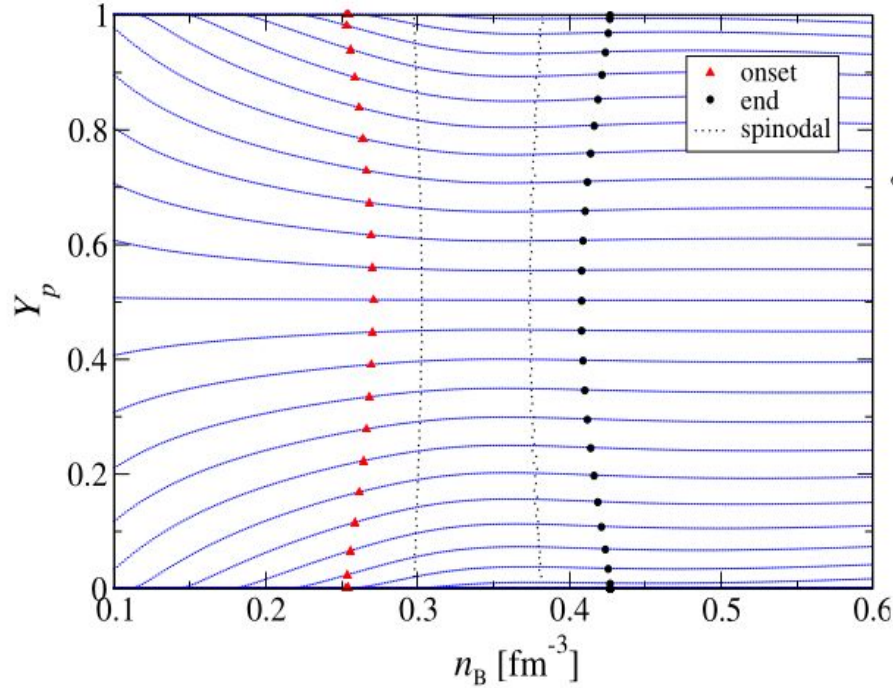
$$g_t(T) = \Theta(T_0 - T) \exp\left[-\frac{t}{2} \left(\frac{T}{T_0 - T}\right)^2\right]$$



Results-I (DD2 EOS with MEV)



Results-II (Phase construction)

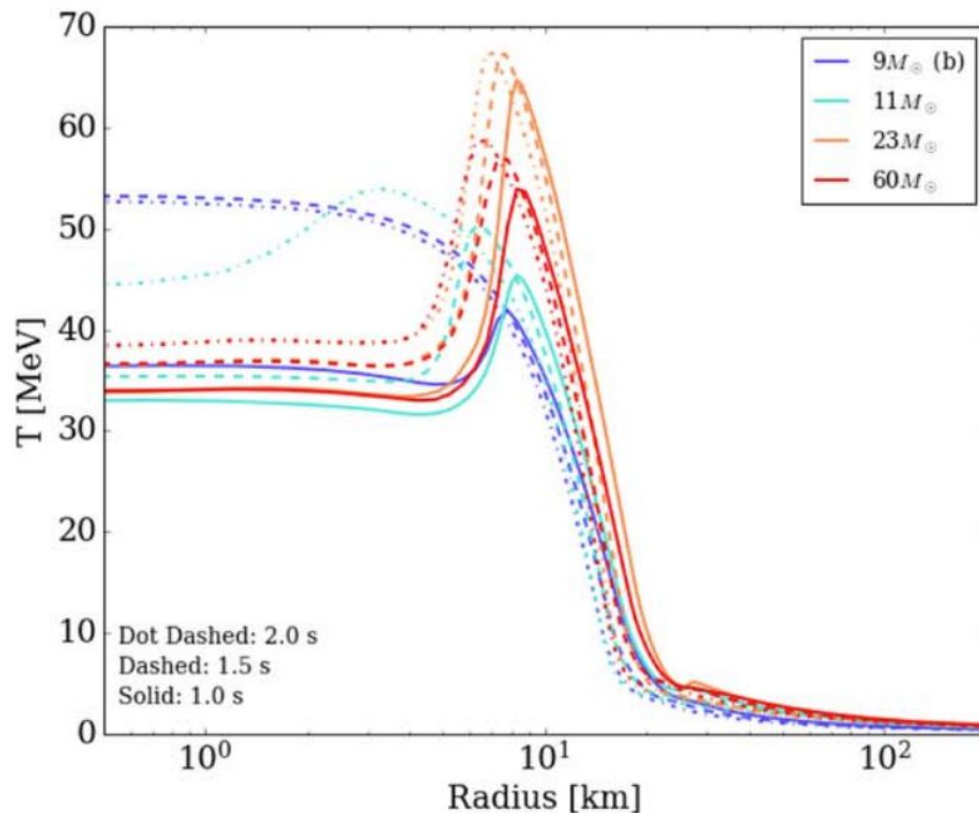


Other hadronic degrees of freedom-Pions

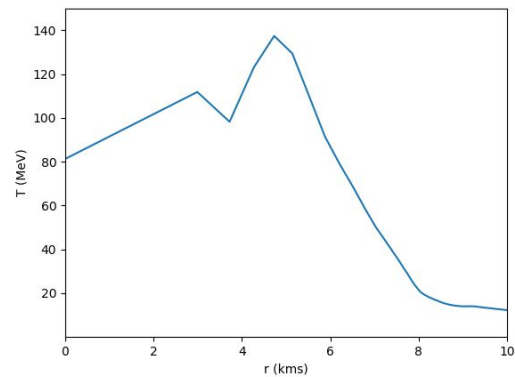
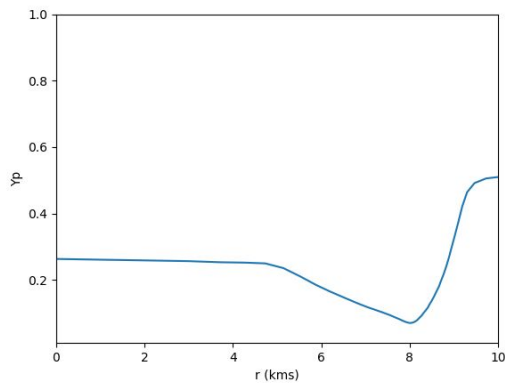
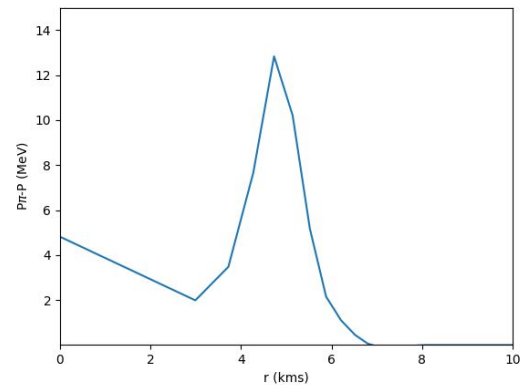
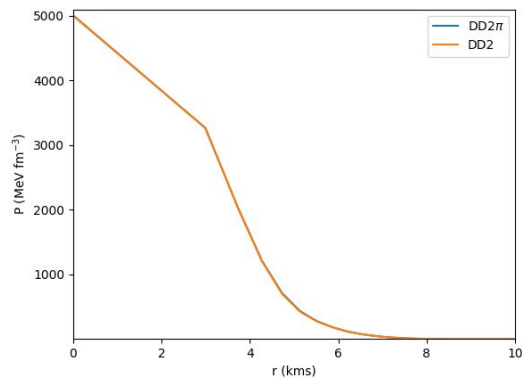
$$\mu_{\pi^-} = -\mu_{\pi^+} = \mu_n - \mu_p$$

$$\mu_{\pi^0} = 0$$

$$n_p + n_{\pi^+} - n_{\pi^-} - n_e = 0$$



Contribution of Pions



State of the Art

interaction between pions and nucleons

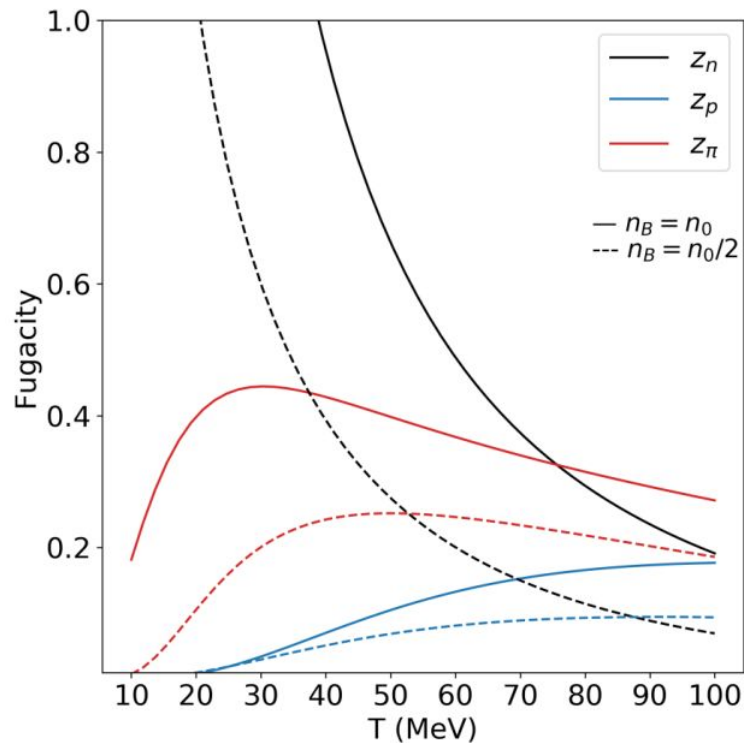
$$\frac{\Omega(T, \{\mu_i\})}{V} = -T \left(\sum_i \frac{b_1^i}{\lambda_i^3} z_i + \sum_{i,j} \frac{b_2^{ij}}{\lambda_i^{3/2} \lambda_j^{3/2}} z_i z_j + \dots \right)$$

$$\text{fugacity} : z_i = \exp \left(\frac{\mu_i - m_i}{kT} \right)$$

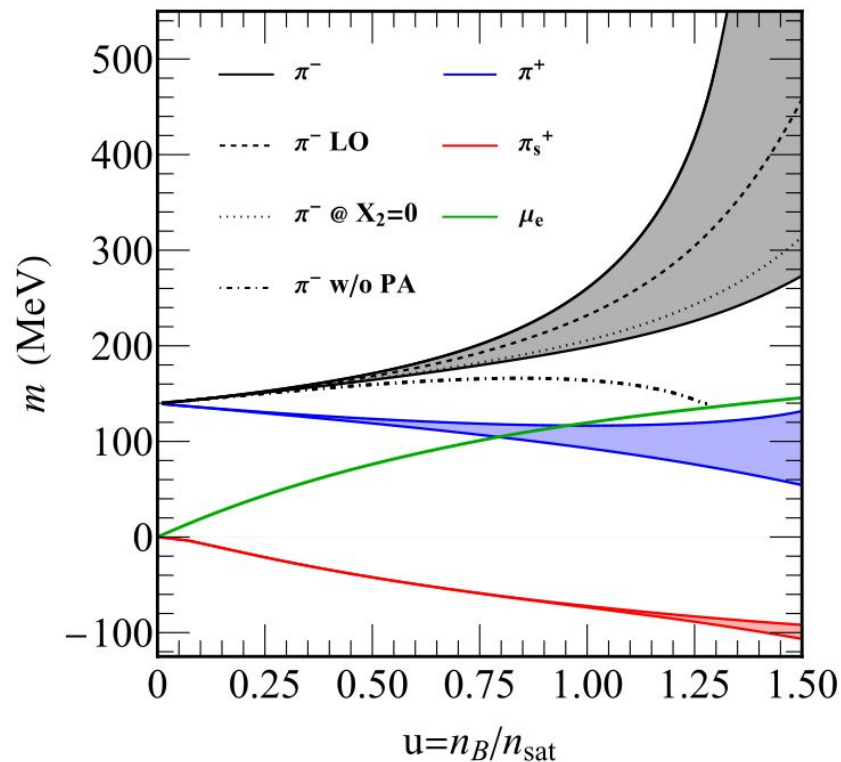
$$b_2^{n\pi^-} = \frac{e^{\beta M}}{2\pi^3} \int_M^\infty dE E^2 K_1(\beta E) \sum_{l,\nu} (2l+1) \delta_{l,\nu}^{3/2}$$

$$b_2^{p\pi^-} = \frac{e^{\beta M}}{2\pi^3} \int_M^\infty dE E^2 K_1(\beta E) \sum_{l,\nu} (2l+1) \left(\delta_{l,\nu}^{3/2} + \delta_{l,\nu}^{1/2} \right)$$

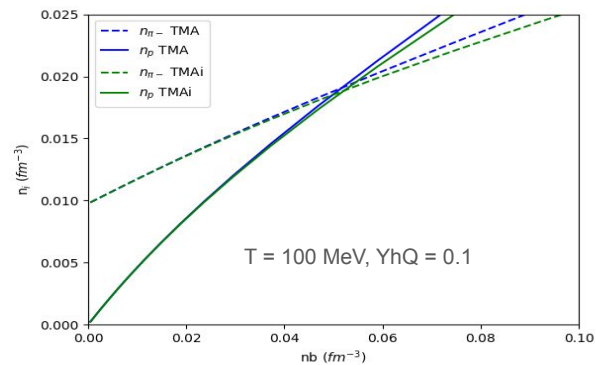
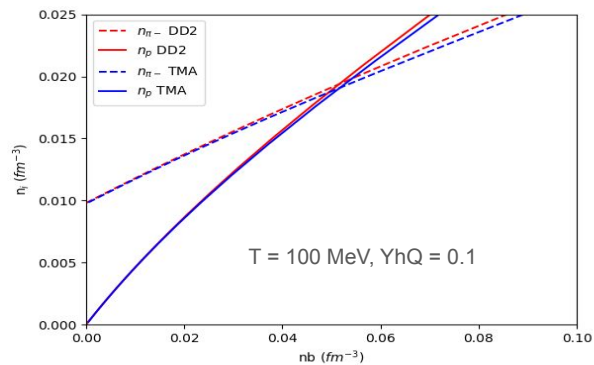
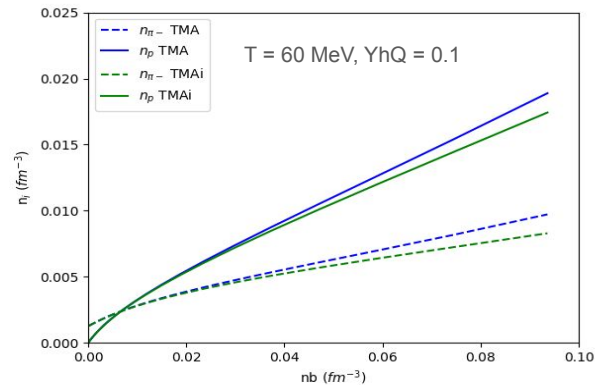
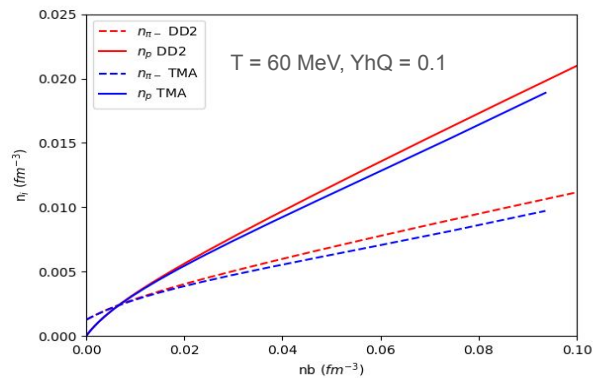
$$P_{\pi^-} = T z_{\pi^-} \int \frac{dk k^2}{2\pi^2} \exp[-\beta \epsilon_\pi(k)] + T z_{\pi^-} \sum_{N=n,p} z_N b_2^{N\pi^-}$$

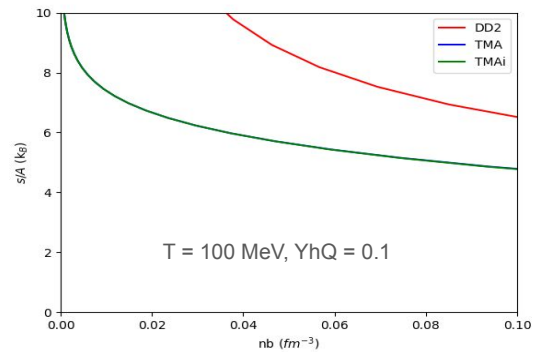
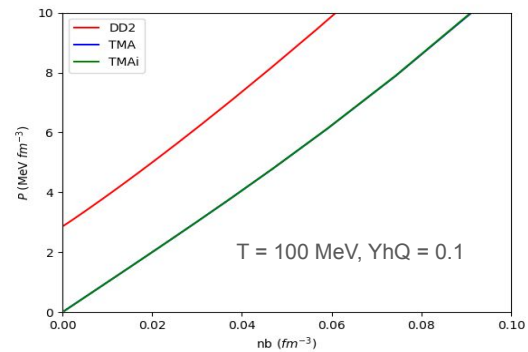
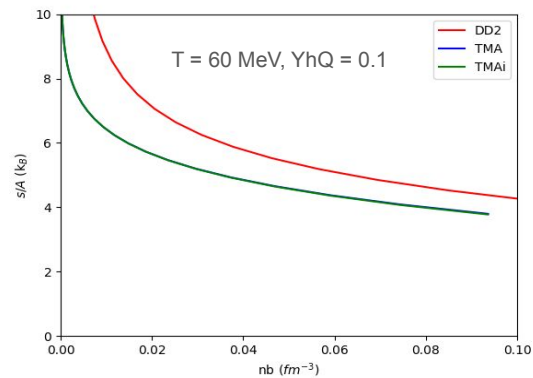
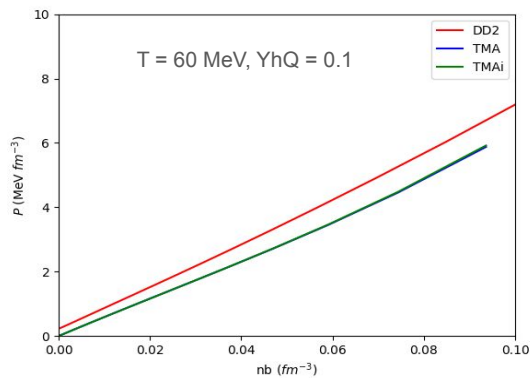


- χ PT calculation of the pion self-energy
- Dressed nucleons account for density-dependent
- Large error bands due to uncertainty in low energy constants from experiments



Pion interaction with RPA





Future perspectives

Pion interaction with nucleons

- Investigate pion–nucleon interactions using RPA with mean-field models
- 3rd virial coefficient

Phase transition

- Other parameters in MEV
- Better quark matter description from non-perturbative QCD

Summary

Supernova simulations are sensitive to QCD phase transition

String flip model needs improvements

EOS with modified excluded volume simplifies the problem

Pion contribution becomes important in high density and high temperature region

There are challenges related to pion interaction